14

WRITTEN TEST

MATHEMATICS

Test Booklet No.

Name of Applicant ..... Answer Sheet No. .....

Application ID : ..... Signature of Applicant : .....

Date of Examination: **.....**

Time of Examination : .....

Signature of the Invigilator(s)

1. .....

2. .....

Duration : 1 Hour] **[Maximum Marks : 50**

IMPORTANT INSTRUCTIONS

(i) The question paper is in the form of Test-Booklet containing **50 (Fifty)** questions. All questions are compulsory. Each question carries four answers marked (A), (B), (C) and (D), out of which only one is correct.

(ii) On receipt of the Test-Booklet (Question Paper), the candidate should immediately check it and ensure that it contains all the pages, i.e., **50** questions. Discrepancy, if any, should be reported by the candidate to the invigilator immediately after receiving the Test-Booklet.

(iii) A separate Answer-Sheet is provided with the Test-Booklet/Question Paper. On this sheet there are **50** rows containing four circles each. One row pertains to one question.

(iv) The candidate should write his/her Application number at the places provided on the cover page of the Test-Booklet/Question Paper and on the Answer-Sheet and NOWHERE ELSE.

(v) No second Test-Booklet/Question Paper and Answer-Sheet will be given to a candidate. The candidates are advised to be careful in handling it and writing the answer on the Answer-Sheet.

(vi) For every correct answer of the question **One (1) mark will be awarded.** For every unattempted question, Zero (0) mark shall be awarded. **There is no Negative Marking.**

(vii) Marking shall be done only on the basis of answers responded on the Answer-Sheet.

(viii) To mark the answer on the Answer-Sheet, candidate should **darken** the appropriate circle in the row of each question with Blue or Black pen.

(ix) For each question only **one** circle should be **darkened** as a mark of the answer adopted by the candidate. If more than one circle for the question are found darkened or with one black circle any other circle carries any mark, the question will be treated as cancelled.

(x) The candidates should not remove any paper from the Test-Booklet/Question Paper. Attempting to remove any paper shall be liable to be punished for use of unfair means.

(xi) Rough work may be done on the blank space provided in the Test-Booklet/Question Paper only.

(xii) *Mobile phones (even in Switch-off mode) and such other communication/programmable devices are not allowed inside the examination hall.*

(xiii) No candidate shall be permitted to leave the examination hall before the expiry of the time.

DO NOT OPEN THIS QUESTION BOOKLET UNTIL ASKED TO DO SO.

1. Which of the following series is not convergent?

(A) $\sum_{n=3}^{\infty} \frac{1}{n^3 \log n}$

(B) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$

(C) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^3}{\underline{|n+1}}$

(D) None of these

2. The infimum and the supremum of the set $\left\{\frac{(-1)^n}{n} : n \in N\right\}$ are respectively

(A) $1, -\frac{1}{2}$

(B) $-1, 0$

(C) $-1, \frac{1}{2}$

(D) None of these

3. Which of the following statement is false?

(A) $f(x) = \sin x$ is not uniformly continuous on $(0, \infty)$

(B) $f(x) = x^2 + 2x + 2$ is uniformly continuous on [1, 2]

(C) $f(x) = \frac{1}{x}$ is not uniformly continuous on (0, 1]

(D) None of these

4. Every bounded sequence has at least one limit point. This represents

(A) Archimedean Property

(B) Heine-Borel Theorem

(C) Bolzano-Weierstrass theorem

(D) Denseness Property

5. Lagrange's mean value theorem is not applicable to the function

(A) $f(x) = e^x$ in [0, 1]

(B) $f(x) = |x+2|$ in [–3, 4]

(C) $f(x) = \cos x$ in $\left[0, \frac{\pi}{2}\right]$

(D) $f(x) = x - 2\sin x$ in $[-\pi, \pi]$

6. Consider the following improper integrals $I = \int_0^{\infty} \frac{\sin^2 x}{x^2} dx$ and $J = \int_1^{\infty} \frac{x^3}{(1+x)^5} dx$, then

(A) Both I and J are divergent

(B) I converges but not J

(C) J converges but not I

(D) Both I and J are convergent

7. The directional derivative of the function $f(x,y,z) = xy^2 + yz^3$ at the point (2, –1, 1) in the direction of $\hat{i} + 2\hat{j} + 2\hat{k}$ is

(A) $\frac{-13}{3}$

(B) $\frac{-11}{3}$

(C) $\frac{-10}{3}$

(D) $\frac{-7}{3}$

8. Which of the following spaces is not compact?

(A) Any closed interval with the usual metric

(B) The discrete space (X, d), Where X is a finite set

(C) The space (R, d), Where R is the set of reals and d is the usual metric

(D) None of these

9. The value of k for which the vector u = (1, k, 5) in V_3 (R) can be expressed as a linear combination of vectors v = (1, –3, 2) and w = (2, –1, 1) is given by :

(A) 3

(B) –8

(C) –2

(D) 4

10. The matrix representing the linear transformation $T : R^2 \to R^2$ defined by $T(x_1, x_2) = (2x_2, 3x_1 - x_2)$ relative to the basis {(1, 3, (2, 5)} is

(A) $\begin{bmatrix} 30 & 48 \\ -18 & -29 \end{bmatrix}$

(B) $\begin{bmatrix} 30 & 48 \\ 18 & 29 \end{bmatrix}$

(C) $\begin{bmatrix} -30 & 48 \\ 18 & -29 \end{bmatrix}$

(D) $\begin{bmatrix} -30 & -48 \\ 18 & 29 \end{bmatrix}$

11. Consider the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$, then

(A) All real-eigen values of the matrix A are positive

(B) All real eigen-values of the matrix A are negative

(C) The matrix A has both positive and negative real eigen-values

(D) The matrix A has no real eigen-values

12. The quadratic form corresponding to symmetric matrix $\begin{bmatrix} 9 & 3 & -3 \\ 3 & 2 & -4 \\ -3 & -4 & 2 \end{bmatrix}$ is

(A) $9x^2 - 2y^2 + 2z^2 - 6xy - 6xz - 8yz$

(B) $9x^2 + 2y^2 - 2z^2 + 6xy + 6xz + 8yz$

(C) $9x^2 + 2y^2 + 2z^2 + 6xy - 6xz - 8yz$

(D) $9x^2 - 2y^2 - 2z^2 + 6xy + 6xz - 8yz$

13. Let $f(x)$ be a minimal polynomial of the 4×4 matrix $A = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

Then the rank of the 4×4 matrix $f(A)$ is

(A) 0

(B) 1

(C) 2

(D) 4

14. The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(\lfloor n)^2}{\lfloor 2n} z^n$ is

(A) 2

(B) 3

(C) 4

(D) 5

15. The harmonic conjugate (v) of the harmonic function $u = y^3 - 3x^2y$ is

(A) $v = -3x^2y + x^3 + c$

(B) $v = -3xy^2 + x^3 + c$

(C) $v = 3xy^2 - x^2 + c$

(D) $v = 3x^2y - x^2 + c$

16. The value of the integral $\int_C \frac{\sin z}{\left(z - \frac{\pi}{4}\right)^3} dz$, where C is $\left|z - \frac{\pi}{4}\right| = \frac{1}{2}$, is equal to

(A) $2\pi i$

(B) 0

(C) $\frac{-\pi i}{\sqrt{2}}$

(D) $\sqrt{2}\pi i$

17. The residue of the function $f(z) = \frac{1}{(z^2+1)^3}$ at $z = i$ is

(A) $\frac{3}{2i}$

(B) $\frac{3}{16i}$

(C) $\frac{4}{3i}$

(D) $\frac{4}{7i}$

18. The coefficient of $\frac{1}{z}$ in the Laurent series expansion of the function $f(z)=\frac{1}{z^2(1-z)}$ about $z = 0$ is

(A) 0

(B) 1

(C) -1

(D) -2

19. In a survey of 200 musicians, it was found that 40 wore gloves on the left hand and 39 wore gloves on the right hand. If 160 wore no gloves at all, then how many wore a glove only on the right hand?

(A) 5

(B) 3

(C) 1

(D) 0

20. The congruence $35x \equiv 14 \pmod{21}$ has

(A) No solution

(B) 5 solutions

(C) 6 solutions

(D) 7 solutions

21. Which of the following groups is not simple?

(A) Group of order 30

(B) Group of order 36

(C) Group of order 40

(D) All of the above

22. Which of the following statement is not correct?

(A) $< x^2 + 1 >$ is a prime ideal of $Z_2[x]$

(B) The polynomial $x^2 + x + 4$ is irreducible over the field of integers modulo 11

(C) $Z[\sqrt{-5}] = \{a + b\sqrt{-5} : a, b \in Z\}$ is an integral domain but not a unique factorization domain

(D) None of these

23. The degree of splitting field of $x^4 - x^2 - 2$ over the field Q of rational numbers, is

(A) 2

(B) 3

(C) 4

(D) 6

24. Which of the following statement is true?

(A) A closed subspace of a normal space is normal

(B) Every locally compact Hausdorff space is regular

(C) Every regular Lindelof space is normal

(D) All of these

25. Let X and Y be two topological spaces and let $f : X \to Y$ be a continuous function. Then

(A) $f(K)$ is connected if $K \subset X$ is connected

(B) $f^{-1}(K)$ is connected if $K \subset Y$ is connected

(C) $f^{-1}(K)$ is compact if $K \subset Y$ is compact

(D) None of these

26. Using method of variation of parameters, the solution of the differential equation $y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$ is

(A) $y = (C_1 + C_2 x)e^{3x} - e^{3x}(\log x + 1)$

(B) $y = (C_1 + C_2 x)e^{3x} + e^{3x}(\log x + 1)$

(C) $y = (C_1 + C_2 x)e^{3x} - e^{3x}(\log x - 1)$

(D) None of these

27. Green's function for the boundary value problem

$\frac{d^2u}{dx^2} = 0, u(0) = u'(1)$ and $u'(0) = u(1)$

is given by

(A) $G(x, \xi) = (2 - \xi)x - \xi, \xi < x \leq 1$

(B) $G(x, \xi) = (1 - \xi)x + 1, \xi < x \leq 1$

(C) $G(x, \xi) = (1 - \xi)x + \xi, \xi < x \leq 1$

(D) None of these

28. The critical point (0, 0) of the system $x'(t) = x - y; y'(t) = x + 5y$ is a

(A) Stable spiral point

(B) Stable centre point

(C) Unstable saddle point

(D) Unstable node point

29. Which of the following is true?

(A) All the eigen-values of a Sturm-Liouville BVP are always natural numbers

(B) All the eigen-values of a Sturm-Liouville BVP are always real

(C) All the eigen-values of a Sturm-Liouville BVP are always in the pair of complex conjugate

(D) None of these

30. The solution of partial differential equation $y^2 p - xyq = x(z - 2y)$ is

(A) $\phi(x + y, yz - y^2) = 0$

(B) $\phi(x^2 - y^2, z + y^2) = 0$

(C) $\phi(x^2 + y^2, yz - y^2) = 0$

(D) None of these

31. The partial differential equation

$$y^2\frac{\partial^2 u}{\partial x^2}-2xy\frac{\partial^2 u}{\partial x\partial y}+x^2\frac{\partial^2 u}{\partial y^2}+2\frac{\partial u}{\partial x}-3u=0$$

is

(A) Parabolic

(B) Elliptic

(C) Hyperbolic

(D) None of these

32. The integral surface for the Cauchy problem $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=1$ which passes through the circle $z = 0$, $x^2 + y^2 = 1$ is

(A) $x^2+y^2+2z^2+2zx-2yz+1=0$

(B) $x^2+y^2+2z^2-2zx-2yz-1=0$

(C) $x^2+y^2+2z^2+2zx+2yz-1=0$

(D) None of these

33. Using Bisection method, a root of equation $x^3-x^2-1=0$ is

(A) 3.136

(B) 2.432

(C) 4.053

(D) 1.46

34. The value of a function $f(x)$ at 5 discrete points is given below :

x :	0	0.1	0.2	0.3	0.4
$f(x)$:	0	10	40	90	160

Using Trapezoidal rule, the value of $\int_0^{0.4} f(x)dx$ is given by

(A) 10.8

(B) 22.0

(C) 38.5

(D) 42.5

35. Given $\frac{dy}{dx}=1-2xy$ with $y(0) = 0$. Then using Euler's method, the value of $y(0.6)$ by taking step size $h = 0.2$ is

(A) 3.1686

(B) 2.1278

(C) 1.4311

(D) 0.5225

36. The Euler's equation corresponding to the functional $\int_0^{\pi}(y'^2-y^2+4y\cos x)dx$ subject to $y(0) = 0$, $y(\pi)=0$ is

(A) $\frac{d^2y}{dx^2}+y=2\cos x$

(B) $\frac{d^2y}{dx^2}+y=2\sin x$

(C) $\frac{d^2y}{dx^2}-y=2\cos x$

(D) None of these

37. The extremal of the functional $I\,[y(x)] = \int_1^2 (y'^2 - 2xy)dx$ subject to $y(1) = 0,\ y(2) = -1$

(A) $y = x^2 - 1$

(B) $y = \frac{1}{6}x(1 - x^2)$

(C) $y = \frac{1}{6}x(7 - x^2)$

(D) None of these

38. The solution of the linear integral equation $\phi(x) = (1+x)^2 + \int_{-1}^{1} (x\xi + x^2\xi^2)\phi(\xi)d\xi$, is

(A) $\phi(x) = 1 + 6x + \frac{25}{9}x^2$

(B) $\phi(x) = 1 - 6x + \frac{5}{9}x^2$

(C) $\phi(x) = 1 + 3x - \frac{25}{9}x^2$

(D) None of these

39. Hamiltonian H is defined as (where L is Lagrangian)

(A) $H = \Sigma q_i p_i^{\bullet} + L$

(B) $H = \Sigma p_i^{\bullet} q_i - L$

(C) $H = \Sigma p_i q_i^{\bullet} - L$

(D) $H = \Sigma p_i^{\bullet} q_i^{\bullet} - L$

40. Let T and V be the kinetic and potential energy of mechanical system, respectively, then the integral $\int_{t_1}^{t_2} (T - V)dt$ has a stationary value. This principle is known as

(A) Hanilton's principle

(B) D' Alembert principle

(C) Principle of least action

(D) None of these

41. The value of λ for which the function $f(x)$ is a valid probability density function, where

$$f(x) = \begin{cases} \lambda(x-1)(2-x), & \text{for } 1 \le x \le 2 \\ 0 \quad , & \text{otherwise} \end{cases}$$

is given by

(A) 5

(B) 6

(C) 7

(D) 1

42. For a certain normal distribution, the first moment about 10 is 40 and the fourth moment about 50 is 48. Then the mean and standard deviation of the distribution respectively are :

(A) 50, 2

(B) 40, 4

(C) 30, 4

(D) 48, 2

43. The moment generating function of Chi-square distribution with n degrees of freedom is

(A) $(1-2it)^{n/2}$

(B) $(1+2t)^{n/2}$

(C) $(1-t)^{-n}$

(D) $(1-2t)^{\frac{-n}{2}}$

44. Any consistent solution of the liklihood equation provides a maximum of the likelihood with

(A) Probability tending to unity as the sample size (n) tends to infinity

(B) Probability tending to infinity as the sample size (n) tends to infinity

(C) Probability tending to zero as the sample size (n) tends to infinity

(D) None of these

45. The mean time to failure (MTTF) for an exponetial distribution with parameter θ is

(A) θ^2

(B) θ

(C) $\frac{1}{\theta}$

(D) $\frac{1}{\theta^2}$

46. The total number of decision variables in the objective function of an assignment problem of size $n \times n$ (n jobs and n machines) is

(A) n^2

(B) n

(C) $2n$

(D) $2n - 1$

47. Suppose that the probability of a dry day following a rainy day is $\frac{2}{3}$ and that the probability of rainy day following a dry day is $\frac{1}{2}$. Given that January 19 is a dry day, what is the probability that January 21 will be a dry day?

(A) $\frac{5}{12}$

(B) $\frac{7}{12}$

(C) $\frac{7}{18}$

(D) $\frac{11}{18}$

48. The minimum value of $z = 20x + 10y$ subject to the constraints : $x + 2y \le 40$; $3x + y \ge 30$; $4x + 3y \ge 60$; $x, y \ge 0$ is

(A) 272

(B) 215

(C) 240

(D) 286

49. Let X(t) be the number of customers in an M / M / 1 queing system with arrival rate $\lambda > 0$ and service rate $\mu > 0$. The process X(t) is a

(A) Poisson process with rate $\lambda - \mu$

(B) Birth and death process with birth rate λ and death rate μ

(C) Pure birth process with birth rate $\lambda - \mu$

(D) Birth and death process with birth rate $\frac{1}{\lambda}$ and death rate $\frac{1}{\mu}$

50. If Tchebycheff's inequality for a random variable X with mean 12 is $P\{6 < X < 18\} \geq \frac{3}{4}$, then the standard deviation of X is

(A) 4

(B) 8

(C) 2

(D) 3

ROUGH WORK