Answer Key: Assistant Professor (Maths)

Q No	Key
1	B
2	B
3	C
4	C
5	B
6	B
7	B
8	B
9	B
10	B
11	A
12	**
13	A
14	A
15	A
16	D
17	D
18	C
19	D
20	**
21	D
22	D
23	A
24	B
25	D

Q No	Key
26	B
27	A
28	C
29	A
30	A
31	B
32	D
33	A
34	A
35	A
36	A
37	C
38	D
39	C
40	C
41	B
42	A
43	B
44	A
45	A
46	C
47	C
48	A
49	D
50	D

** Moved out of assessment

Test Booklet Series

A

Test Booklet No.

Test Booklet for the Post of
Assistant Professor Mathematics

Name of Applicant .... Answer Sheet No. ....

Applicant ID/Roll No. : .... Signature of Applicant : ....

Date of Examination: ....

Signature of the Invigilator(s)

1. ....

Time of Examination : ....

2. ....

Duration : 1½ Hours] **[Maximum Marks : 50**

IMPORTANT INSTRUCTIONS

(i) The question paper is in the form of Test-Booklet containing **50 (Fifty)** questions. All questions are compulsory. Each question carries four answers marked (A), (B), (C) and (D), out of which only one is correct. Choose the correct option or the most appropriate option.

(ii) On receipt of the Test-Booklet (Question Paper), the candidate should immediately check it and ensure that it contains all the pages, i.e., **50** questions. Discrepancy, if any, should be reported by the candidate to the invigilator immediately after receiving the Test-Booklet.

(iii) A separate Answer-Sheet is provided with the Test-Booklet/Question Paper. On this sheet there are **50** rows containing four circles each. One row pertains to one question.

(iv) The candidate should write his/her Application ID/Roll number at the places provided on the cover page of the Test-Booklet/Question Paper and on the Answer-Sheet and NOWHERE ELSE.

(v) No second Test-Booklet/Question Paper and Answer-Sheet will be given to a candidate. The candidates are advised to be careful in handling it and writing the answer on the Answer-Sheet.

(vi) For every correct answer of the question **One (1) mark will be awarded.** There will be negative marking and 1/4 (0.25) mark will be deducted for every incorrect answer.

(vii) Marking shall be done only on the basis of answers responded on the Answer-Sheet.

(viii) To mark the answer on the Answer-Sheet, candidate should **darken** the appropriate circle in the row of each question with Blue or Black pen.

(ix) For each question only **one** circle should be **darkened** as a mark of the answer adopted by the candidate. If more than one circle for the question are found darkened or with one black circle any other circle carries any mark, the answer will be treated as incorrect.

(x) The candidates should not remove any paper from the Test-Booklet/Question Paper. Attempting to remove any paper shall be liable to be punished for use of unfair means.

(xi) Rough work may be done on the blank space provided in the Test-Booklet/Question Paper only.

(xii) *Mobile phones (even in Switch-off mode) and such other communication/programmable devices are not allowed inside the examination hall.*

(xiii) No candidate shall be permitted to leave the examination hall before the expiry of the time.

DO NOT OPEN THIS QUESTION BOOKLET UNTIL ASKED TO DO SO.

1. The number of elements of order 11 in a group of order 33 is

(A) 0 (B) 10

(C) 20 (D) 30

2. The number of conjugate classes of non-Abelian group of order 27 is

(A) 8 (B) 11

(C) 12 (D) 9

3. The isomorphism class of the group (GF(64)/GF(2)) is

(A) S_3 (B) $Z_3 \oplus Z_3$

(C) Z_6 (D) None of these

4. A homomorphism from a simple group is

(A) Trivial but not one-to-one (B) Non trivial but one-to-one

(C) Either trivial or one-to-one (D) None of these

5. Let R be a commutative ring with unity and I be a prime ideal of R then which of the following is true?

(A) I must be maximal ideal of R

(B) If R/I is finite ring then I is maximal ideal

(C) I is never maximal

(D) If I is maximal ideal then R/I must be finite ring

6. The number of zero divisor in a commutative ring with unity $(Z_{20}, +_{20}, \times_{20})$ is

(A) 8 (B) 11

(C) 9 (D) 13

7. The solution of $xdy = [x^3(x^2 + y^2) + y]\, dx$ is

(A) $\tan^{-1}(y/x) = x^3/3 + c$ (B) $\tan^{-1}(y/x) = x^4/4 + c$

(C) $\tan^{-1}(x/y) = x^4/4 + c$ (D) $\tan^{-1}(x/y) = x^3/3 + c$

8. The singular solution of $\left(x\frac{dy}{dx} - y\right)^2 = \left(\frac{dy}{dx}\right)^2 - 1$ is

(A) $x^2 + y^2 = 1$ (B) $x^2 - y^2 = 1$

(C) $x^2 + y^2 = 2$ (D) $x^2 - y^2 = 2$

9. The eigen valve for boundary value problem $\frac{d^2y}{dx^2} + \lambda y = 0$ with boundary condition $y(0) = 0$, $y(\pi) + y'(\pi) = 0$ satisfy

(A) $\lambda + \tan \lambda\pi = 0$ (B) $\sqrt{\lambda} + \tan\sqrt{\lambda}\pi = 0$

(C) $\sqrt{\lambda} + \tan \lambda\pi = 0$ (D) $\lambda + \tan\sqrt{\lambda}\pi = 0$

10. For the diffusion problem $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}(0 < x < \pi, t > 0)$ with boundary conditions $u(0, t) = 0 = u(\pi, t)$, $u(x, 0) = 3 \sin 2x$, then it's solution is given by $u(x, t) =$

(A) $3e^{-t} \sin 2x$ (B) $3e^{-4t} \sin 2x$

(C) $3e^{-9t} \sin 2x$ (D) $3e^{-2t} \sin 3x$

11. If the partial differential equation $(x-1)^2\frac{\partial^2 u}{\partial x^2}-(y-2)^2\frac{\partial^2 u}{\partial y^2}+2x\frac{\partial u}{\partial y}+2xyu=0$ is parabolic in $S \subseteq R^2$ but not in $R^2 - \{S\}$. Then, S is

(A) $\{(x, y) \in R^2 ; x = 1 \text{ or } y = 2\}$

(B) $\{(x, y) \in R^2 ; x = 1\}$

(C) $\{(x, y) \in R^2 ; x = 1 \text{ and } y = 2\}$

(D) $\{(x, y) \in R^2 ; y = 2\}$

12. The differential equation $\frac{dy}{dx}=1+y^2, y(0)$ in the domain R : $|x| < 5, |y| < 3$ has

(A) No solution

(B) Unique solution for $|x| < 0.3$

(C) Infinite number of solution

(D) Nothing about solution can be concluded

13. The feasible region of linear programming problem is always

(A) Polytope

(B) Convex Polyhedron

(C) Hyperplane

(D) None

14. The number of basic solution to the system

$$2x_1 + x_2 - x_3 + x_4 + 2x_5 = 6$$

$$4x_1 + 3x_2 + x_3 + 2x_4 = 12$$

is

(A) 9

(B) 12

(C) 6

(D) 3

15. In the queuing system arrival process is

(A) Poisson's process

(B) Exponentially process

(C) Geometrically distributed

(D) None of these

16. The dual problem of linear programming problem max $Z = -2x_1 + x_2$ s.t. $x_1 - x_2 \leq 10$, $2x_1 - x_2 \leq 40$, $x_1, x_2 \geq 0$ has

(A) Feasible solution

(B) Optimum solution

(C) Unbounded solution

(D) None of these

17. Which one of the following is true?

(A) Every linear programming problem has a feasible solution

(B) If a linear programming problem has an optimum solution then it is unique

(C) The union of two convex sets is necessarily convex

(D) Extreme points of the disk $x^2 + y^2 \leq 1$ are the point on the circle $x^2 + y^2 = 1$

18. In a testing of hypothesis problem, the density of a sufficient statistic T is $f(t, \theta) = \frac{\theta}{t^{\theta+1}}$, $t > 1,\ \theta > 0$. The hypothesis $H_0 : \theta = 1$ against $H_1 : \theta = 2$ is to be tested and T = 2.5 is observed. Then, the p-value of the most powerful test is

(A) 0.05

(B) 0.5

(C) 0.6

(D) 0.4

19. Suppose X is random variable with E(X)= Var(X). Then, the distribution of X is

(A) Necessarily poisson

(B) Necessarily normal

(C) Necessarily exponential

(D) Cannot be identified from the given data

20. If $P(A \cap B) = 0$, then $P\left(\frac{A}{P \cup B}\right)$ = ?

(A) $\frac{P(A)}{P(B)}$

(B) $\frac{P(A)}{P(B) + P(A \cup B)}$

(C) $\frac{P(A)}{P(A) + P(B)}$

(D) $\frac{P(B)}{P(A) + P(B)}$

21. Let V be the space of all $n \times n$ matrices and T : V $\rightarrow$ V be a linear operator defined by $T(A) = \frac{A + A^T}{2}$, then nullity of T is

(A) $2n$

(B) $n^2/2$

(C) $\frac{n(n+1)}{2}$

(D) $\frac{n(n-1)}{2}$

22. Consider the system of linear equations $x + y + z = 3$, $x - y - z = 4$, $x - 5y + kz = 6$. Then, the value of k, for which this system has an infinite number of solutions is

(A) $k = 0$

(B) $k = -1$

(C) $k = -3$

(D) $k = -5$

23. Let $M_1 = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, $M_2 = \begin{bmatrix} -3 & 0 \\ 1 & -3 \end{bmatrix}$ and $M_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, then the minimal polynomial of the matrix $\begin{bmatrix} M_1 & 0 & 0 \\ 0 & M_2 & 0 \\ 0 & 0 & M_3 \end{bmatrix}$ is

(A) $x^2(x - 3)^3(x + 3)^2$

(B) $x^2(x^2 - 9)$

(C) $x^2(x^2 - 9)^2$

(D) $x(x - 3)^4(x + 3)^2$

24. The transformation $T : R^3 \to R^2$ defined by $T(x, y, z) = (x + y, y + z)$ is

(A) Linear and has zero kernel

(B) Linear and has a proper subspace as kernel

(C) Neither linear nor one-one

(D) Neither linear nor onto

25. The dimension of Hom (C^3, R^2) is

(A) 6

(B) 18

(C) 12

(D) Does not exist

26. The function $f(x) = \frac{1}{2}\left(\frac{|\sin x|}{\cos x} + \frac{\sin x}{|\cos x|}\right)$ is a periodic with period

(A) π

(B) 2π

(C) $\pi / 2$

(D) None of these

27. If A, U and P denote the set of Absolutely convergent, uniformly convergent and pointwise convergent sequences respectively, then which of the following is correct containment

(A) $A \subset U \subset P$ (B) $P \subset A \subset U$

(C) $P \subset U \subset A$ (D) Does not exist

28. The series $\sum_{n=0}^{\infty} \frac{1}{4}\{1+(-1)^{n+1}(2n+1)\}$ is

(A) Convergent (B) Divergent

(C) Oscillating infinitely (D) Oscillating finetely

29. A function $f(x)=\begin{cases} x^{\alpha}\sin\frac{1}{x^{\beta}} & x \neq 0 \\ 0 & x=0 \end{cases}$ is bounded variation in [0, 1] if

(A) $\alpha > \beta$ (B) $\beta > \alpha$

(C) $\alpha = \beta$ (D) None of these

30. If $<a_n>$ = 1 – $\frac{(-1)^n}{n}$, then the limit of its telescopic series $\Sigma(a_{n+1}-a_n)$ is

(A) 1 (B) 0

(C) –1 (D) 2

31. The infinite series $\sum \frac{(-1)^{n-1}\sin nx}{n^2}$ is

(A) Divergent

(B) Absolutely convergent

(C) Oscillatory

(D) Convergent but not absolutely convergent

32. Which is not correct about the Cantor ternary set?

(A) It is not closed

(B) It is uncountable

(C) It is perfect set

(D) It is dense

33. If $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = \log 2$, then $\sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)}$ is equal to

(A) 1 – 2 log 2

(B) 1 + log 2

(C) $(\log 2)^2$

(D) $-(\log 2)^2$

34. The domain of function $f(x) = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

(A) [–2, 0) ⋃ (0, 1)

(B) [–2, 1)

(C) (–2, 0) ⋃ (0, 1)

(D) (–2, 0) ⋃ [0, 1)

35. The branch point of multivalued function $f(z) = \tan^{-1}(z^2 + 2z)$ is given by

(A) $z^2 + 2z + i = 0$

(B) $z^2 - 2z - i = 0$

(C) $z^2 - 2z + i = 0$

(D) None of these

36. If $f(z) = \frac{z^2 + 5z + 6}{z-2}$ and the path of integration is a circle with originated center and radius r the Cauchy's theorem is applicable, whenever r equals to

(A) 1

(B) 2

(C) 3

(D) 1 and 2

37. The value of the integral $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-4)(z-2)} dz$ where C is the circle $|z| = 3$ traced anticlock-wise is

(A) $-2\pi i$ (B) $i\pi$

(C) $-i\pi$ (D) $2i\pi$

38. A complex function $f(z) = \begin{cases} e^{-1/z^2} & z \neq 0 \\ 0 & z = 0 \end{cases}$ then

(A) $f(z)$ is continuous at $z = 0$

(B) $f(z)$ is differentiable at $z = 0$

(C) $f(z)$ is analytic at $z = 0$

(D) $f(z)$ has essential singularity at $z = 0$

39. If a bilinear transformation $W = \frac{az+b}{cz+d}$ which has one fixed point at α and the other fixed point ∞, has the form $\omega - \alpha = \lambda(z - \alpha)$, then λ equal to

(A) d/a (B) c/b

(C) a/d (D) b/c

40. The value of λ for which integral equation $y(x) = \lambda \int_0^1 (6x - t) y(t)\, dt$ are given by root of equation

(A) $(3\lambda - 1)(2 + \lambda) - \lambda^2 = 0$ (B) $(3\lambda - 1)(2 + \lambda) + 2 = 0$

(C) $(3\lambda - 1)(2 + \lambda) - 4\lambda^2 = 0$ (D) $(3\lambda - 1)(2 + \lambda) + \lambda^2 = 0$

41. The resolvent kernel of integral equation $y(x) = f(x) + \int_{\log 2}^{x} e^{t-x} y(t)dt$ is given by

(A) $\cos(x - t)$

(B) 1

(C) 2

(D) e^{t-x}

42. The Integral equation $y(x) = \int_{0}^{x} (x-t)y(t)dt - x\int_{0}^{1} (1-t)y(t)dt$ is equivalent to

(A) $y'' - y = 0, \; y(0) = 0, \; y(1) = 0$

(B) $y'' - y = 0, \; y(0) = 0, \; y'(0) = 0$

(C) $y'' + y = 0, \; y(0) = 0, \; y(1) = 0$

(D) $y'' + y = 0, \; y(0) = 0, \; y'(0) = 0$

43. For a continuous function $f(t)$, $0 \leq t \leq 1$, the integral equation $y(t) = f(t) + 3\int_{0}^{1} ts\, y(s)\, ds$ has

(A) Unique solution if $\int_{0}^{1} s\, f(s)\, ds \neq 0$

(B) Infinetly many solution if $\int_{0}^{1} s\, f(s)\, ds = 0$

(C) No solution if $\int_{0}^{1} s\, f(s)\, ds = 0$

(D) Infinitely many solution if $\int_{0}^{1} s\, f(s)\, ds \neq 0$

44. Degree of freedom is

(A) The minimum number of independent coordinates required to specify the system

(B) The maximum number of independent coordinates required to specify the system

(C) The minimum number of dependent coordinates required to specify the system

(D) The maximum number of dependent coordinates required to specify the system

45. The Lagrange's equation for conservative system are

(A) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = 0$

(B) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial \dot{q}_k} = 0$

(C) $\frac{d}{dt}\left(\frac{\partial L}{\partial q_k}\right) - \frac{\partial L}{\partial q_k} = 0$

(D) None of the above

46. Newton-Raphson iteration formula for finding $\sqrt[3]{c}$ where $c > 0$ is

(A) $x_{n+1} = \frac{2x_n^3 + \sqrt[3]{c}}{2x_n^2}$

(B) $x_{n+1} = \frac{3x_n^3 - \sqrt[3]{c}}{3x_n^2}$

(C) $x_{n+1} = \frac{2x_n^3 + c}{3x_n^2}$

(D) $x_{n+1} = \frac{2x_n^3 - c}{3x_n^2}$

47. Which of the following statement is correct?

(A) The number of positive real roots of $P_n(x) = 0$ cannot exceed the number of changes of sign of $P_n(x)$

(B) The number of negative real roots of $P_n(x) = 0$ cannot exceed the number of changes of sign of $P_n(-x)$

(C) Both are true

(D) None of these

48. The functional $\int_0^1 (y'^2 + x^3)\, dx$, $y(1) = 1$ has

(A) Strong minima on all of its extremal

(B) Strong maxima on all of its extremal

(C) Weak maxima on all of its extremal

(D) Weak minima on all of its extremal

49. The variational problem of extremizing the functional $I[y(x)] = \int_1^3 y(3x - y)dx$; $y(3) = 4\frac{1}{2}$, $y(1) = 1$ has

(A) An unique solution

(B) Exactly two solution

(C) An infinite number of solution

(D) No solution

50. Consider the functional $I(y) = \int_a^b F(y, y')dx$; $y' = dy/dx$, $y(a) = y_1, y(b) = y_2$ where $y \in c^2[a, b]$, F has second order continuous partial derivatives with respect to y, y' and y_1, y_2 are given real number. Let $y = y(x)$ be an extremizing function for the functional I. Then, along the extremizing curve

(A) $F(y, y') = C$

(B) $\frac{\partial F}{\partial y} = 0$

(C) $F - y \frac{\partial F}{\partial y'} = C$

(D) $F - y' \frac{\partial F}{\partial y'} = C$

ROUGH WORK

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